Analytic Spectral Integration of Birefringence-Induced Iridescence

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Birefringence

Birefringence—also known as “double refraction”—is an optical phenomenon where the perceived refractive-index depends on the light’s polarization and direction.
Light travelling in a birefringent medium is decomposed into two linearly polarized rays.

Birefringence

Arises...

- naturally in crystals of non-cubic lattice systems due to the anisotropic electric properties of the crystal lattice;
- artificially via an external electric or magnetic field;
- by the material's supra-molecular structure—what is known as "form birefringence";
- via "photoelasticity", i.e. induced by deformations due to mechanical stress.

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Very different mechanisms, same consequence: *Optical anisotropy*.

Why do we care?

- Can be perceived in everyday objects: Glass under stress, moulded plastics, crystals and gemstones, etc.

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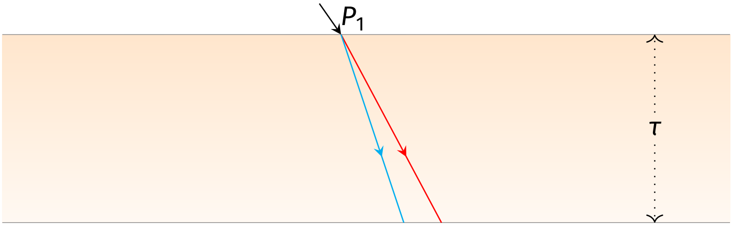
- Can be perceived in everyday objects: Glass under stress, moulded plastics, crystals and gemstones, etc.
- Essential to a wide range of practical applications: Liquid crystals, polarized-phase microscopy, medical diagnostics, polarized light imaging (PLI), experimental physics, etc.

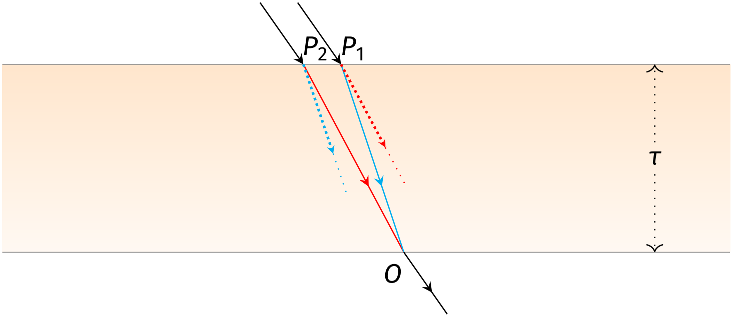
Optical Anisotropy

Light refracting into such media is split into two linearly polarized rays, an **ordinary** and an **extraordinary** ray. The extraordinary ray behaves in a peculiar manner, is detached from its wavevector and ignores Snell's law of refraction and the law of reflection.



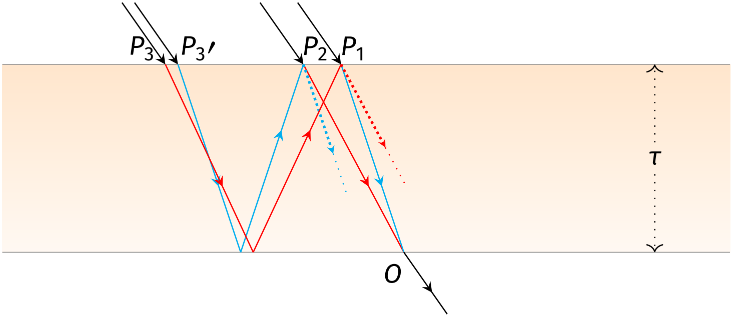
Iridescence

Constructive-destructive interference arises once light refracts through optically anisotropic media due to the velocity differences between the ordinary and extraordinary rays.



Iridescence

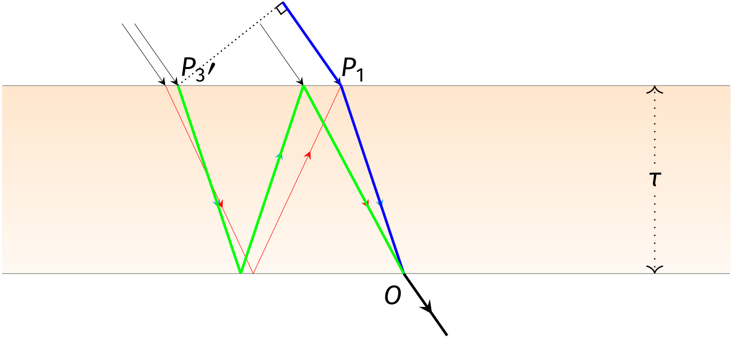
Light that undergoes internal reflection should be taken into account as well.
2^{2N+1} distinct paths that undergo $2N$ internal-reflections.



Iridescence

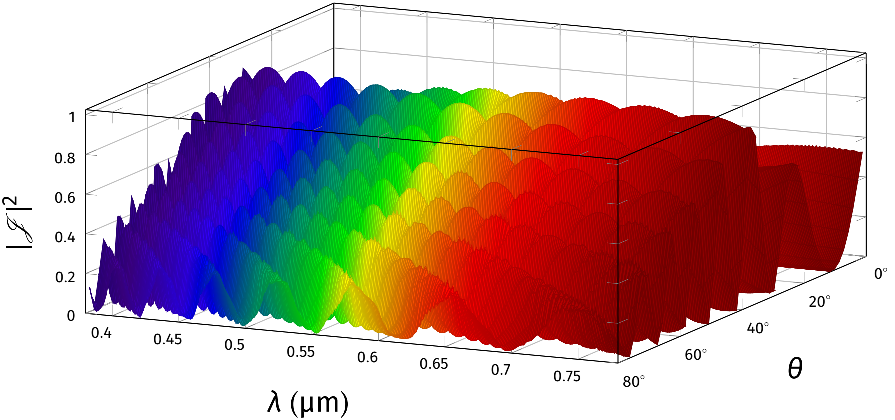
Iridescence effects arise because:

- Constructive-destructive interference due to phase differences between paths.
- Change of polarization due to retardation.



Spectral Integration - Challenges

Spectral aliasing — integrand oscillates very rapidly!



$\eta_o = 1.44$, $\eta_e = 1.42$, $A = [\,0.25, 0.96, 0.13\,]^T$, $\tau = 0.50$ mm, s-polarized incidence light

Spectral Integration

$$\mathbf{s} = \int_{\lambda} \left| \sum_k E_k \, e^{i\Delta\phi_k} \right|^2 \Lambda(\lambda) \, d\lambda$$

- $\left| \sum_k E_k e^{i\Delta\phi_k} \right|^2$ — Intensity of aggregated complex amplitudes.
- E_k — Peak amplitude.
- $e^{i\Delta\phi_k}$ — Phase shift.
- $\Lambda(\lambda)$ — Support function. Colour matching and/or emission spectrum, etc.

Spectral Integration

In a similar manner to [**Belcour:17**], the integral can be rewritten as:

$$\mathbf{S} = \int_{\boldsymbol{\lambda}} \left| \sum_k E_k e^{i\Delta\phi_k} \right|^2 \Lambda(\lambda) d\lambda =$$

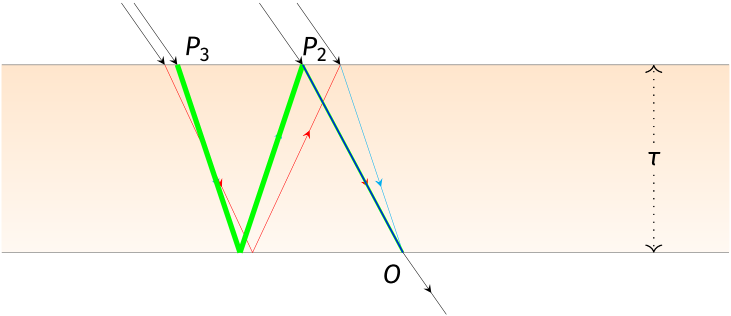
$$= \sum_j E_j^2 + 2 \sum_j \sum_{l=1}^{\infty} E_j E_{j+l} \int_{\boldsymbol{\lambda}} \cos\left(\Delta\phi_j - \Delta\phi_{j+l}\right) \Lambda(\lambda) d\lambda =$$

$$= \mathcal{E} + 2\,\mathcal{H}$$

- $\mathcal{E}$ — The transmissivity.
- $\mathcal{H}$ — Pair-wise constructive-destructive interference.

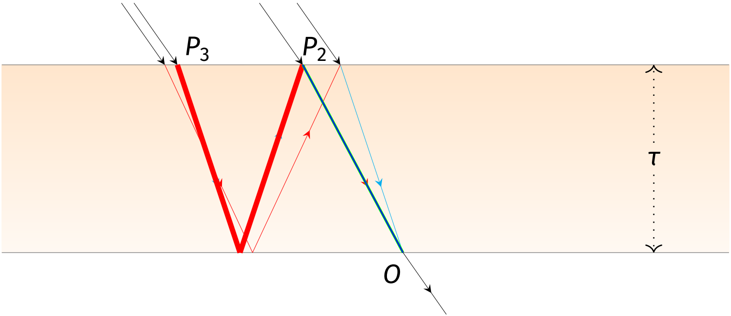
Approximating the interference contribution

- The optical path length of a path is the sum of the optical path lengths of all the segments that compose it.



Approximating the interference contribution

- The optical path length of a path is the sum of the optical path lengths of all the segments that compose it.
- Therefore, the optical path difference between the green and the blue paths below is the optical path length of the red path.



Approximating the interference contribution

$$\mathcal{H} = \sum_j \sum_{l=1}^{\infty} E_j E_{j+l} \int_{\boldsymbol{\lambda}} \cos\left(\Delta\phi_j - \Delta\phi_{j+l}\right) \Lambda(\lambda) d\lambda$$

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$$\approx \sum_j \sum_{l=1}^{\infty} E_j E_{j+l} \int_{\boldsymbol{\lambda}} \cos\left(\Delta\phi_l\right) \Lambda(\lambda) d\lambda$$

$$= \left(\sum_j E_j^2 C \right) \cdot \left(\sum_{l=1}^{\infty} E_l \Re\left\{ \int_{\boldsymbol{\lambda}} e^{i\Delta\phi_l} \Lambda(\lambda) d\lambda \right\} \right)$$

See the paper for more details.

Approximating the interference contribution

$$\mathcal{H} = \boxed{\left(\sum_j E_j^2 C\right)} \cdot \boxed{\left(\sum_{l=1}^{\infty} E_l \Re\left\{\int_{\boldsymbol{\lambda}} e^{i\Delta\phi_l} \Lambda(\lambda) d\lambda\right\}\right)} =$$

$$= \lim_{M\to+\infty} \boxed{\left(\sum_j E_j^2 C\right)} \cdot \boxed{\left(\sum_{m=1}^{M} \sum_{l}^{2^{2M}} E_l \Re\left\{\int_{\boldsymbol{\lambda}} e^{i\Delta\phi_l} \Lambda(\lambda) d\lambda\right\}\right)}$$

With M being is the maximal count of double internal-reflections of the paths' "heads". Setting M to 1 is almost always sufficient.

- Energy decreases exponentially as a function of internal-reflection count.

Approximating the interference contribution

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- Energy decreases exponentially as a function of internal-reflection count.
- Loss of optical coherence.

Results

Rendered at about 1ms/frame on a mobile GeForce 1070 at a resolution of 1920x1080.

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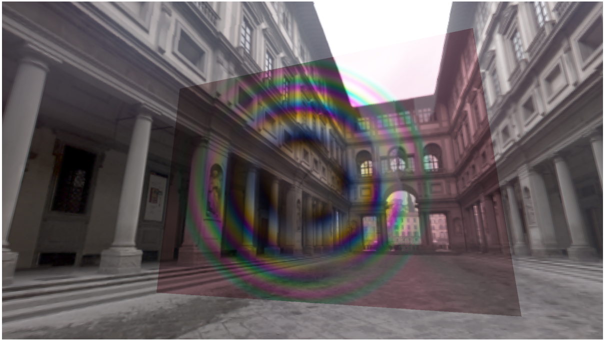
See the paper for an extensive evaluation, as well as the implementation included with the supplemental material.

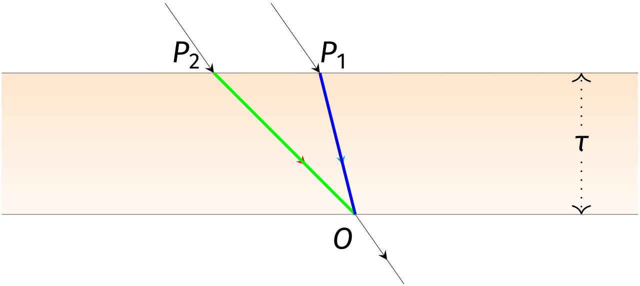
Optical Coherence

A wave ensemble from an infinitely small light source with an infinitely precise spectral line would be fully coherent, i.e. perfectly spatially and temporally correlated.
Physical light sources are, however, neither. E.g. filtered sunlight has a coherence size of about 60 μm [**Mashaal:12**].



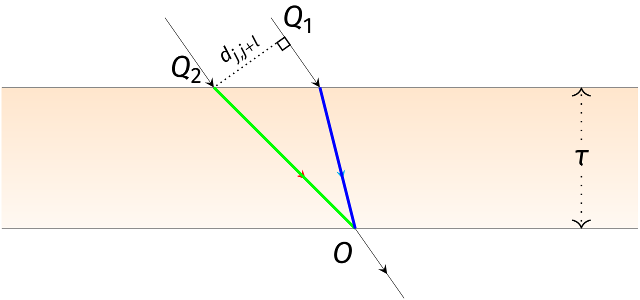
Optical Coherence

As before, suppose a wavefront is refracted into our slab at two points and then recombines on exit. The intensity of the resulting wave ensemble is a superposition of the time-and-ensemble averaged contributions of the two rays once they refract out. [**Wolf1995ocqo.book**]



Optical Coherence

This is captured by $\gamma\left(d_{j,j+l}, t_j - t_{j+l}\right) \in [0,1]$, the *normalized mutual coherence function*. [**Wolf1995ocqo.book**]
Where $d_{j,j+l}$ is distance on the wavefront, and $t_j - t_{j+l}$ is the time difference.



Optical Coherence

Then $\mathcal{H} = \sum_j \sum_{l=1}^{\infty} E_j E_{j+l} \gamma\left(d_{j,j+l}, t_j - t_{j+l}\right) \int_{\boldsymbol{\lambda}} \cos\left(\Delta\phi_j - \Delta\phi_{j+l}\right) \Lambda(\lambda) d\lambda.$

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When the wave ensemble is uncorrelated, $\mathcal{H}$ vanishes.

$$\mathbf{S}_{incoherent} = \mathcal{E}$$

Optical Coherence

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When the wave ensemble is uncorrelated, $\mathcal{H}$ vanishes.

$$\mathbf{S}_{incoherent} = \mathcal{E}$$

When the wave ensemble is fully coherent

$$\mathbf{S}_{coherent} = \mathcal{E} + 2\mathcal{H}$$

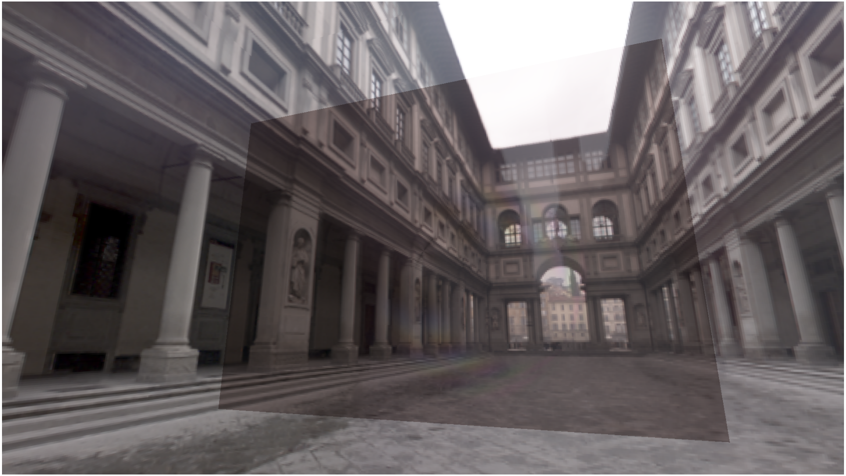
Optical Coherence

Coherence size of 0.20 μm

Optical Coherence

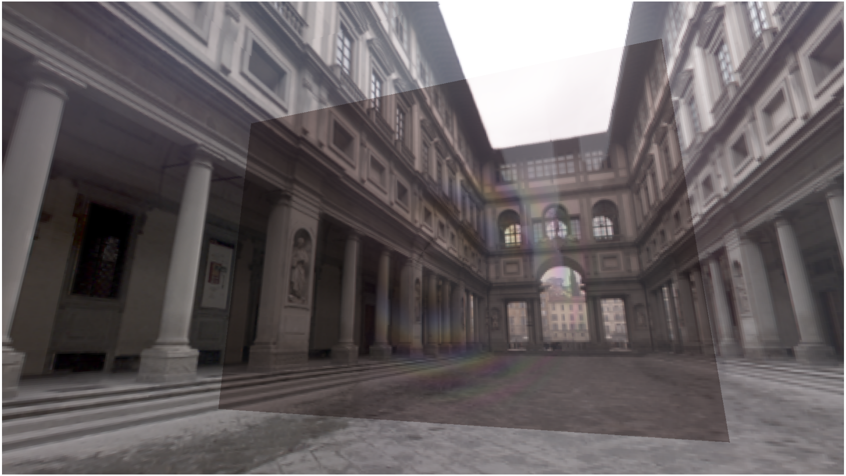
Coherence size of 2 µm

Optical Coherence



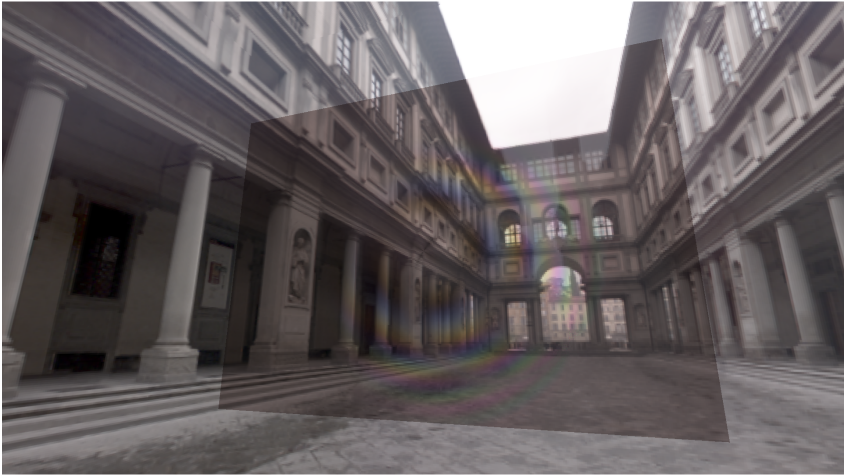
Coherence size of 4 µm

Optical Coherence



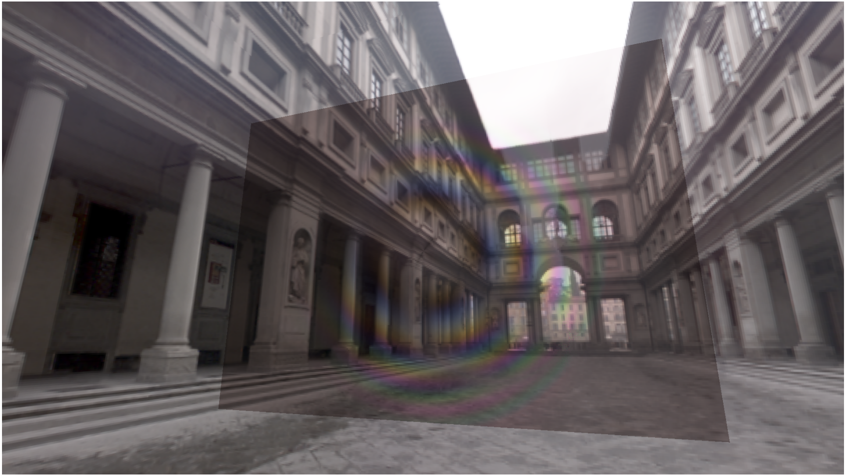
Coherence size of 6 μm

Optical Coherence



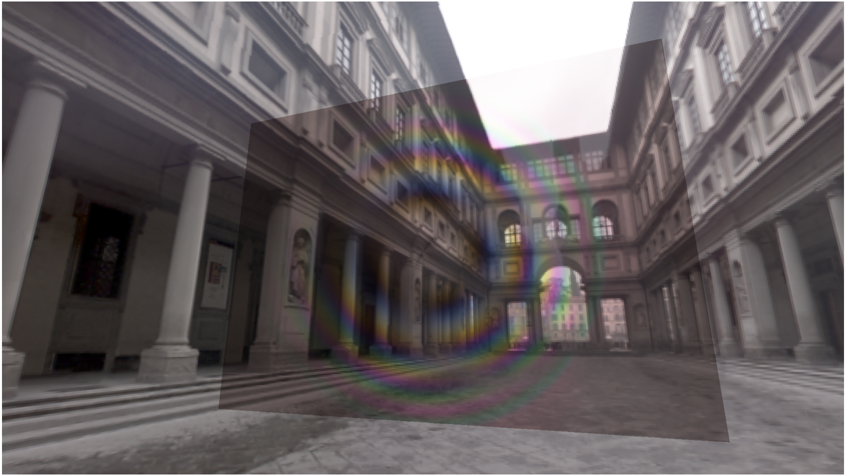
Coherence size of 10 µm

Optical Coherence



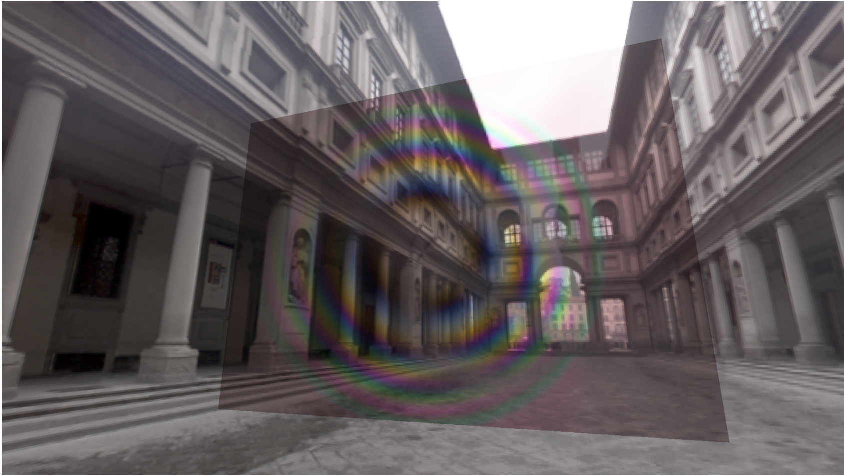
Coherence size of 15 μm

Optical Coherence



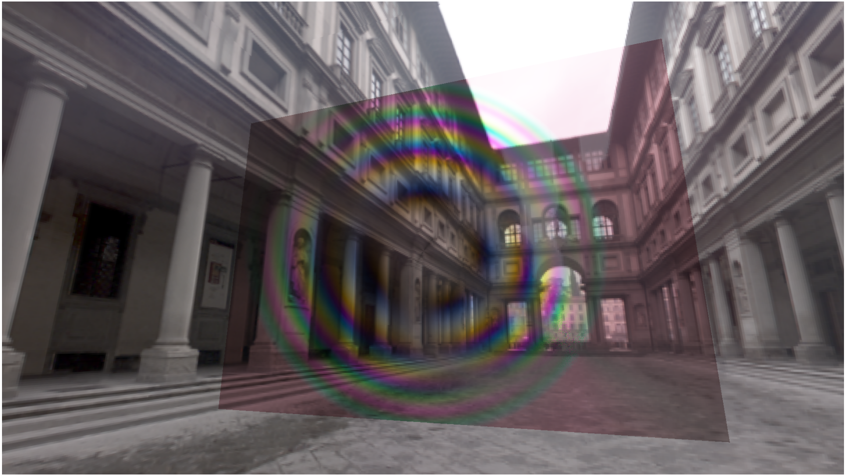
Coherence size of 20 μm

Optical Coherence



Coherence size of 30 µm

Optical Coherence



Coherence size of 200 µm

Thank you
for your attention!

Additional Slides